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Modelling of a relative income tax bracket-based progression with the effect of a slower tax burden growth

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ABSTRACT
This study aims to model the distribution of the tax burden in schedular progressive taxation and to describe the key characteristics of such models, in particular their differences from the models based on continuously increasing smooth functions of the relationship between the tax burden and the taxpayer's income. Our hypothesis is that the use of the Gompertz function to model the main indicators of tax burden distribution of the schedular progressive income tax will help us approximate and formalize the distribution of the tax burden in a relative income tax bracket-based progression. Our research relies on the hypothetico-deductive model, more specifically, on mathematical hypothesis testing. The methodological framework comprises models of progressive taxation and mathematical methods, including data approximation based on the use of the Gompertz function, analysis of the antiderivative and convexity of functions and their properties. The resulting model can be used to describe the dynamic characteristics of the relationship between the tax burden and certain parameters of schedular taxation. This model can help identify the level of income beyond which the progression of the tax burden becomes formal and does not generate commensurately high revenue growth. The existence of such income level results in what can be considered the key drawback of the relative progression in question – the impossibility to provide a significant difference (step) of the tax burden progression in the whole interval of the taxpayer's income. What makes this research practically significant is that the proposed methodology allows us to take into account the actual tax burden in modelling the parameters of the relative progression.

KEYWORDS
income tax; progressive scale; schedule; tax rates; Gompertz function

JEL H24, J31, O15

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Моделирование относительной поразрядной прогрессии подоходного налога с эффектом замедления роста налоговой нагрузки

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АННОТАЦИЯ
Цель исследования заключается в моделировании распределения налогового бремени при шедулярном прогрессивном налогообложении, выявление основных свойств данных моделей и их отличий от моделей, основанных на

применении непрерывно возрастающих гладких функций зависимости налоговой нагрузки от величины доходов налогоплательщиков. Гипотеза исследования заключается в том, что использование функции Гомпертца для моделирования основных показателей распределения налогового бремени шедулярного прогрессивного подоходного налога позволит аппроксимировать и формализовать распределение налогового бремени при поразрядной относительной прогрессии подоходного налога. Процедура исследования опирается на использование гипотетической дедуктивной модели, в частности, на проверку математической гипотезы. Методологической базой исследования являются модели реализации прогрессивного налогообложения и математические методы, в том числе аппроксимация данных с использованием функции Гомпертца, методы анализа свойств первообразной и выпуклости функций. Полученная модель может быть использована для описания динамических характеристик взаимосвязи между налоговой нагрузкой и некоторыми параметрами шедулярного налогообложения. Эта модель может помочь определить уровень дохода, при превышении которого увеличение налогового бремени становится формальным и не приводит к соизмеримому высокому росту доходов. Существование такого уровня доходов обуславливает главный недостаток применения поразрядной относительной прогрессии подоходного налога – невозможность обеспечения существенного шага прогрессии налоговой нагрузки на всем интервале доходов налогоплательщиков. Практическая значимость результатов заключается в разработке методологии, позволяющей учитывать фактическую налоговую нагрузку при моделировании параметров поразрядной относительной прогрессии подоходного налога.

КЛЮЧЕВЫЕ СЛОВА
подоходный налог; прогрессивная шкала; шедулярный подход; налоговые ставки; функция Гомпертца

1. Introduction

Unlike proportional tax models, which differ only in terms of tax rates, non-taxable income and tax deductions as well as all the national models of progressive taxation are unique. Every state sets the parameters of their progressive tax systems following the national traditions, social and economic goals and objectives.

There are two fundamentally different approaches to modelling progressive taxation: global and schedular. The former relies on the use of continuously increasing smooth functions of the relationship between the tax burden and the taxpayer's income. When the global approach is applied, one tax is imposed on all the income, regardless of its nature. The schedular approach is widely used in practice. Different schedules can be taxed at different tax rates (for example, in Russia the schedule of earned income and the schedule of unearned income) and even in different ways – proportionally or progressively.

The fact that schedular progressive taxation systems that use a relative bracket-based progression are widely spread makes this method of taxation a topic worthy of research interest. Despite the existing evidence of applying a relative progression in taxation, there is still no uniform approach to its modelling. Moreover, there is still a perceived lack of a well-supported theoretical rationale for the available practical solutions and ways of ensuring vertical equity.

This study aims to address these gaps by modelling the distribution of the tax burden in schedular progressive taxation and describing the key characteristics of these models, in particular their differences from the models based on continuously increasing smooth functions of the relationship between the tax burden and the taxpayer's income.

Our hypothesis is that the use of the Gompertz function to model the main indicators of tax burden distribution of the schedular progressive income tax will help us approximate and formalize the distribution of the tax burden in a relative income tax bracket-based progression.

Formalization of the model of a relative progression can reveal the key characteristics of this taxation method and show the threshold level of income beyond which the progression of the tax burden becomes formal and does not generate commensurately high tax revenue growth.

2. Literature review

Income taxation resides at the core of the secondary distribution of income, which can be approached from different perspectives. The main questions that need to be considered in this respect is how to make income taxation more fair and how the chosen taxation model will affect people's labour and business motivation as well as the situation in the public sphere.

The largest body of research on income taxation focuses on the most important question – the fairness of progressive taxation, more specifically, whether the rich pay should more in taxes than the poor (see, for example, Popescu et al. [1], Chambers et al. [2], Krajewski et al. [3], and Mirrlees [4]).

The impact of tax rates on labour supply is another question, which, despite its importance, remains largely underexplored although the research by Luksic [5] and Kireenko et al. [6] suggests that progressive tax scales do not have a negative effect on people's sentiments and motivation. Nevertheless, these matters still require a more detailed in-depth research.

In general, the distribution of the income taxation burden (through progressive or proportional taxation) has a certain effect on the social sphere, positive as well as negative. It is, therefore, interesting to estimate the impact of progressive taxation on the happiness of A citizens. Such study was conducted by Oishi et al. [7], who found that progressive tax burden distribution has a positive influence on citizens' happiness levels in the USA. This effect, however, was not similar for all social groups as the wealthiest Americans demonstrated a slight decrease in happiness.

Due to the heterogeneity of public interests, increased scholarly attention is paid to political aspects of progressive income taxation (see, for example, Garcia-Muniesa [8], Carriero et al. [9], Oh [10], and Mehrotra [11]).

Another area of research is the analysis of the practical issues of tax reforms such as the adoption or abolition of progressive income tax scales (see, for example, Barrios et al. [12], Balatsky et al. [13], Di Nola et al. [14], Vlad et al. [15], Mayburov [16], and Hyun et al. [17]). The cases of particular countries are of most interest since they provide sufficient evidence to verify many of the theoretically justified hypotheses. One of such hypotheses is that progressive income taxation is associated with higher rates of tax evasion.

The impact of the tax burden on income tax evasion still remains a widely discussed problem in the theory of taxation (see, for example, Holter et al. [18], Belozyorov et al. [19], and Landier et al. [20]).

Other significant areas of research include studies of the impact of progressive income taxation on economy and the social sphere. Despite the substantial research evidence accumulated in each of these areas, there still remain issues for debate.

Studies of the organization of progressive taxation can be roughly divided into two large groups: studies of the first group analyze the international experience of progressive taxation and the influence of specific solutions on economy and the social sphere (see, for example, Stephenson [21] and Musgrave et al. [22]); studies of the second group are based on modelling progressive income tax scales (see, for example, Mirrlees [4], Chistyakov et al. [23], Kim [24], Smirnov

[25], and Saez [26]). This way the existing systems of income taxation can be improved or adjusted to specific needs of national economies and societies.

Our study places a special emphasis on the mathematical analysis of the most widely spread schedular model of progressive taxation.

Chistyakov et al. [23] proposed a game-theoretical model of the optimal scale of average rates of the progressive income tax. A distinguishing feature of this model is that it does not require to take into account the function of income distribution, which means that the impact of other mandatory payments on the model in question can be excluded.

Kim [24] developed various models of progressive income taxation based on the global approach. In comparison with schedular progressive taxation, such models have their own advantages and shortcomings, which, however, fall beyond the scope of our research.

Saez [26] proposed a methodology to determine the optimal tax rate of the non-linear income tax scale for high income based on the elasticity of taxable income. His research [26] develops the optimal income tax formulae proposed by Mirrlees [4], which also described the impact of tax burden on national economy.

This study does not consider the impact of the income tax on economy. Modeling of income tax scales based on income elasticity, although crucial for decision-making in taxation, does not take into account the factors of vertical equity and social stratification. In our view, such models can complement other models that take into account these factors.

Our methodology is based on the use of the Gompertz function, which is applied in other fields of research such as biology [27; 28], medicine [29], geoscience [30; 31], demography [32], agriculture [33; 34], zootechnics [35] and economics [36]. In economics, the Gompertz curves were used to analyze social stratification depending on income distribution [36].

All of the above leads us to the following conclusion. There is a substantial body of research literature on the fairness of progressive taxation, optimal tax rates, the influence of tax burden on labour and the social sphere, the effect of public interests on income taxation, specific cases of income taxation reforms, the impact of income tax rates on compliance, modelling of tax scales and the application of the Gompertz function to describe various empirical relationships. There are, however, no studies that would use the Gompertz curve to analyze the relative income tax bracket-based progression.

3. Modelling of a relative income tax bracket-based progression

The methodology for calculating tax burden distribution proposed in this study relies on the income data, which also take into account the burden from other taxes and levies. Thus, we can exclude the negative influence of the distortions in the estimates of real disposable income caused by the co-existence of several taxes and levies within the tax system.

Our research relies on the hypothetico-deductive model, or, to be more precise, on the most widely spread form of this model – mathematical hypothesis.

Methodologically, the study is based on the use of the Gompertz function, analysis of properties of the antiderivative and convexity of the functions and other methods of function analysis.

The Gompertz function is a sigmoid function and a special case of the generalized logistic function. Gompertz functions are defined and continuous in the entire interval; the first and second derivative exist on each point and are finite. A peculiar feature of the Gompertz function is that it belongs to the type of mathematical models describing a pattern of growth that is the slowest at the beginning and at the end of the interval. Growth slows down at a lower pace than it accelerates [37].

Our analysis covers the average and marginal tax rates, which are defined as follows:

- average (actual) tax rate or tax burden is the ratio of the amount of taxes paid (T) to the total tax base x:

$$N(x) = \frac{T}{x}; \tag{1}$$

- marginal tax rate $r_{0...i...n}$ is the rate specified in the legislative act on this tax, that is, it is the tax rate established under the corresponding law [38].

The model of tax burden distribution in the schedular progressive tax system is normative since it has a system of limitations stemming from the principles of schedular progressive taxation.

The problem with using the Gompertz function to model the distribution of the tax burden in a schedular progressive tax system is that the function does not equal zero if the argument is zero while the tax cannot be paid in the absence of income. Therefore, to address our research task, we used the antiderivative of the Gompertz function and the domain of the function was limited only to positive values of the argument.

To choose the methodology for calculating the tax burden, we need to take into account the needs of the real economy. Taxpayers' net income should increase together with the growth in their nominal income:

$$\frac{x_i - r_i(x_i)}{x_{i-1} - r_i(x_{i-1})} > 1 \text{ for } x_i > x_{i-1}, \tag{2}$$

where x_i is the income level in the i^{th} step; r_i is the rate of the tax payable in the i^{th} step.

This principle means that there should be no jump discontinuities in the income that would otherwise occur if the effective tax rate was raised disproportionately to the taxpayer's income.

This condition can be met in two ways:

a) the marginal tax rates increase continuously throughout the whole interval of the taxpayer's income;

b) through the use of schedular taxation: the amount of tax is calculated on a cumulative basis. In the case of a higher tax rate, only the sum exceeding the threshold is taxed.

Each of these solutions has its advantages and drawbacks and requires to follow other principles that stem from social needs and the nature of progressive income taxation:

c) higher income is taxed at a higher rate;

d) zero income means the absence of tax burden.

While one can choose either condition (a) or (b), conditions (c) and (d) are mandatory for any model of progressive income taxation.

In addition, the following considerations should be taken into account:

1) Condition (a) will be met by any continuous smooth function. Conditions (c) and (d) are met if the function of the effective tax rate increases continuously from zero. This condition is met if

$$\frac{dy}{dx} > 0,\ x > 0 \text{ and } y < 1.$$

These conditions are also referred to as the conditions of normalcy of the progressive scale [23]. The main advantage of this method of progressive taxation is that there is no need to set income intervals (schedules).

2) The schedular method (b) is widely used in the practice of progressive taxation because it implies simpler calculations.

In schedular progressive taxation, tax is calculated on a cumulative basis as the sum of products of parts of the taxpayer's income multiplied by the marginal tax rates for corresponding income ranges (3).

$$T(x)\begin{cases} x > 0 \\ T_0(x) = r_0 x \\ T_i(x) = r_i(x - a_{i-1}) + T_{i-1}, \\ r_i \in (0,1),\ i = \overline{0;n} \\ 0 < a_0 < a_{i-1} < a_i < a_n, \end{cases} \tag{3}$$

where n is the number of income groups; x is the share of the taxpayer's actual income assigned to the i^{th} group; a_i is the marginal income for the corresponding tax rate; r_i is the i^{th} tax rate $T(x)$ is the amount of tax levied on income x.

4. Modeling of the tax burden and marginal tax rates depending on the level of the taxpayer's nominal income

The actual tax burden $N(x_i)$ of a taxpayer with the income (x_i) exceeding a_n in this model of progressive taxation will look the following way:

$$N(x) = \frac{T(x)}{x} = \frac{r_0 a_0 + \sum_{i \in n} r_i (x - a_{i-1})}{x}. \quad (4)$$

Marginal tax rates r_i can be represented by the following model:

$$f(r) = \begin{cases} r_0,\ T(x) \in 0 \ldots r_0 a_0 \\ r_i,\ T(x) \in a_{i-1} \ldots r_i a_i \\ \ldots \\ r_n,\ T(x) \in a_{n-1} \ldots r_n a_n. \\ 0 < a_0 < a_{i-1} < a_i < a_n \\ r_0 < r_i < r_n \\ r_i \in (0,\ 1),\ i = \overline{0;\ n} \end{cases} \quad (5)$$

The system of limitations (5) may be described by the threshold piecewise continuous function of the relationship between the marginal tax rates and the taxpayer's income level. A stepwise increase in the tax rates results in the growth of the tax burden in the entire interval of the taxpayer's income.

At the origin of the coordinates, the marginal tax rate equals tax burden $N_0 = r_0$. Further, tax burden increases in the intervals from a_{i-1} to a_i. In other words, the growth of the tax burden is behind the growth of the marginal tax rate by the share of the tax burden corresponding to lower tax rates.

As the income grows, the portion of income taxed at lower rates becomes smaller $T(x) \to r_i \cdot x$, $N(x) \to r_n$ for $x \to \infty$. This leads to slower growth of the tax burden, which comes close to the level of the highest marginal tax rate but will always remain below this level because there is always a part of the tax burden stemming from lower marginal tax rates. For lower income earners, the tax burden grows slower than for middle income earners. For higher income levels, the share of the income taxable at lower rates is less noticeable in the total tax base and the growth of the tax burden slows down. On drawing near the maximum marginal tax rate, the growth of the tax burden slows down.

Thus, the functional relationship between the tax burden and the taxpayer's nominal income is to a great extent similar to Gompertz curves and is characterized by the following:

- $0 \le f(x) \le r_n$;
- $f(x) = 0$ for $x = 0$;
- $f(x) > 0$ for $x > 0$;
- the function is continuous and strictly increasing in the entire interval $x > 0$;
- $\lim_{x \to \infty} f(x) = r_n$.

The relationship between the tax burden and the taxpayer's income described with the help of the Gompertz function looks the following way:

$$f(x) = r_n \cdot e^{b \cdot e^{(c \cdot x)}}, \quad (6)$$

where b and c are negative.

The function has horizontal asymptotes determined by the following formulae:

$$\lim_{x \to -\infty} r_n \cdot e^{b \cdot e^{c \cdot x}} = 0$$

$$\text{and} \quad (7)$$

$$\lim_{x \to \infty} r_n \cdot e^{b \cdot e^{c \cdot x}} = r_n \cdot e^0 = r_n.$$

The curve approaches the asymptotes asymetrically. In accordance with the supposition that the tax burden takes the form of a Gompertz curve, the tax rate approaches r_n when the income (x) approaches ∞.

Parameter b determines the shift of the tax burden curve along the axis of taxable income (x). Its value depends on the breakpoint of the first schedule or the amount of tax-free personal allowance. The higher is the value of this parameter, the longer is the interval of the taxpayer's income preceding the beginning of the interval with the high rate of growth in the tax burden's dependence on the taxpayer's income. A lower value of parameter b signifies higher financial security enjoyed by minimum income citizens.

However, at $x = 0$ the Gompertz function takes the value $f(0) = r_0 \cdot e^b$, which corresponds to the case where the taxpayer with zero income would still bear the minimum tax burden and thus the key principle of income taxation would be compromised (d).

Since the Gompertz function is differentiable in the entire interval and any continuous function has an infinite number of antiderivatives $F(x) + \text{const}$, we are going to use the following antiderivative as a scale of average rates for zero tax burden:

$$N(x) = r_n \cdot e^{b \cdot e^{c \cdot x}} - r_n \cdot e^b. \qquad (8)$$

Function $N(x)$ will equal zero for $x = 0$ and the asymptote will change by the value of the constant. Fig. 1 shows examples of graphs of the Gompertz function and its antiderivative for constant values $r_n = 0.4$, $b = –4$, $c = –0.5$. For dimensionality reduction along the x-axis, linear data normalization is used with the fiducial value of 1 million roubles (Fig. 1).

The value of x where the function reaches the value equal to half of asymptote ($f(x) = r_n / 2$) is determined as

$$x = \frac{\ln\left(\frac{\ln(2)}{-b}\right)}{c}.$$

5. Modelling the level of income characterized by slower growth of the tax burden

The function of the relationship between the tax burden and income is progressive in the entire income interval due to the function's asymmetry and tendency towards the maximum marginal tax rate. However, the step of the progression decreases gradually. The tax burden becomes less and less progressive. For a certain high level of income, the taxpayer's income is to a significant extent liable to the maximum marginal tax rate while the difference between the amount of the tax burden and the maximum marginal tax rate becomes insignificant.

Although there is a certain amount of subjectivity involved in deciding what constitutes a significant or insignificant difference, mathematical analysis enables us to identify the level of income x_p beyond which the growth of the tax burden will start to slow down. In fact,

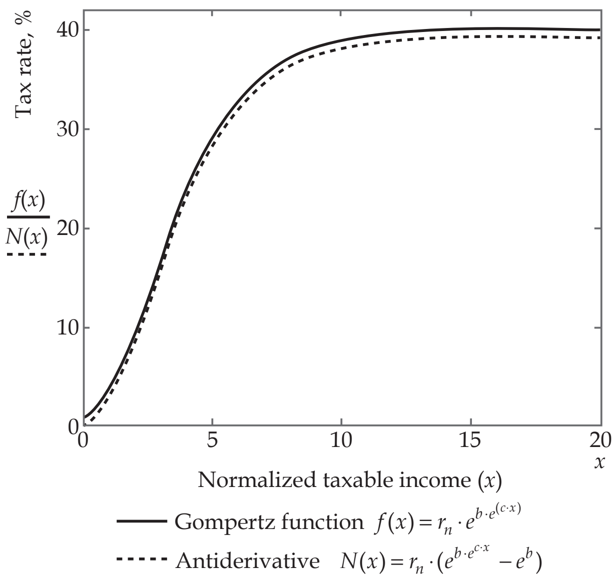

***Fig. 1.* Gompertz function $f(x) = r_n \cdot e^{b \cdot e^{(c \cdot x)}}$ and antiderivative $N(x) = r_n \cdot (e^{b \cdot e^{c \cdot x}} - e^b)$**

Source: authors' calculations

x_p is the point of inflection of function $N(x)$. The income beyond this level will provide a less significant increment in the tax burden and, therefore, tax progression will become formal and will not result in commensurately higher government revenue.

To find income x_p, we are going to find the point of inflection of function $N(x)$. It is known that if the second derivative in the point in the given interval changes sign, then this point is the function's point of inflection. For a double-continuously differentiable Gompertz function, to find the point of inflection it is enough to find such value of x that the second derivative will take zero value.

To analyze the Gompertz function, we need to find its first and second derivative.

$$f'(x) = r_n \cdot b \cdot c \cdot e^{b \cdot e^{(c \cdot x)} + c \cdot x}, \quad (9)$$

$$f''(x) = r_n \cdot b \cdot c^2 \cdot (b \cdot e^{c \cdot x} + 1) \cdot e^{b \cdot e^{c \cdot x} + c \cdot x}. \quad (10)$$

Evidently, the second derivative equals zero if one of the factors equals zero. In our case, $b \cdot e^{c \cdot x} + 1 = 0$.

Therefore, in interval $0 \leq x < +\infty$, the Gompertz function is absolutely continuous and has a point of inflectionat

$$x_p = \frac{\ln\left(-\frac{1}{b}\right)}{c}. \quad (11)$$

Let us now look at the properties of the convexity of the curve.

In interval

$$0 \leq x < \frac{\ln\left(\frac{-1}{b}\right)}{c},$$

the first derivative is positive and increasing and the second derivative is also positive. Thus, the function is convex down. This means that within the given interval, there is a progressive dependence of the tax burden on the taxpayer's income. The higher is the income, the more substantial is the tax burden.

After the inflexion point, the first derivative is positive and the second derivative is negative, the function is increasing more and more slowly and the curve is convex upwards. In this interval, a higher income does not generate a proportional increment in tax revenue. The progression of the tax burden gradually decreases (Fig. 2).

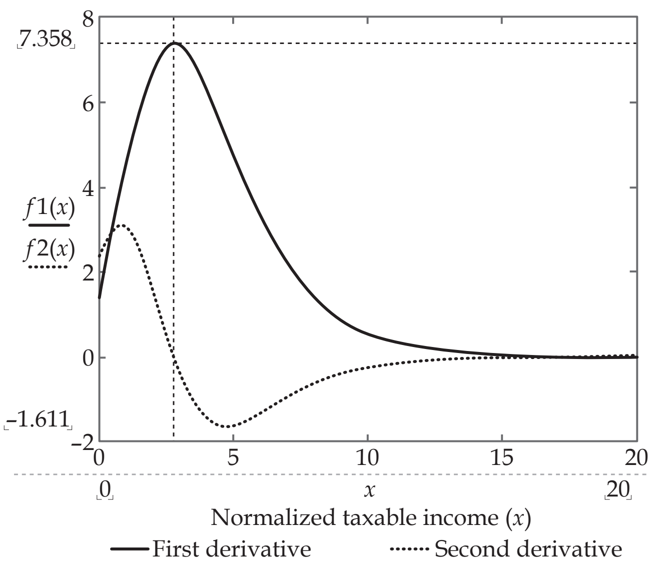

***Fig. 2.* First and second derivatives of the function of the relationship between the tax burden and the taxpayer's income**

Source: authors' calculations

We know that the growth rate of the function, or the relative rate of change of the function, equals the logarithmic derivative of the function.

The growth rate of the function is defined as

$$\frac{d}{dx}\ln(f(x))$$

or

$$temp(x) = \frac{1}{f(x)} \cdot \left(\frac{d}{dx} f(x)\right).$$

Thus, for the Gompertz function, the rate of growth will change in accordance with formula $temp(x) = b \cdot c \cdot e^{c \cdot x}$ and the choice of coefficients *b* and *c* determines the curve of the function's growth.

The graph in Fig. 3 shows the rate of increase of the function with coefficients *b* and *c* held constant.

At $x = 0$ the growth rate equals $temp = b \cdot c$.

The growth rate drops by half at

$$x = \frac{-\ln(b \cdot c)}{c}$$

(in the given case we take Δx = 1.386). After that, the growth slows down.

Fig. 4 shows the graph of surfaces of 10 antiderivative curves when changing parameter *c* from –5 to –0.5 with the step of 0.5.

The flexion of the surface of the curves along the *z*-axis shows the influence of coefficient *c* on the graph of the relationship between the tax burden and income. Coefficient *c* determines the growth rate of the function. The value of *c* depends on the ratio of intervals (schedules) and the step of the increment (difference) of the values of the marginal tax rates: the faster is the maximum marginal tax rate reached, the higher is the step. In the case of tax burden distribution, this parameter shows how pronounced is the progressivity of the income tax rates. The goals set by the national government in particular circumstances can thus be met effectively through the regulation of the parameters of a progressive tax system.

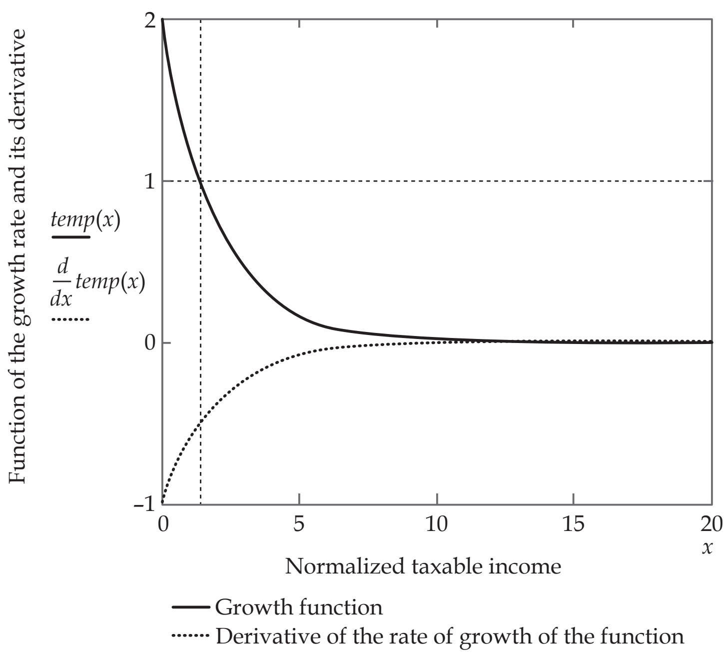

***Fig. 3.* Graph of the rate of growth of the function with coefficients *b* and *c* held constant**

Source: authors' calculations

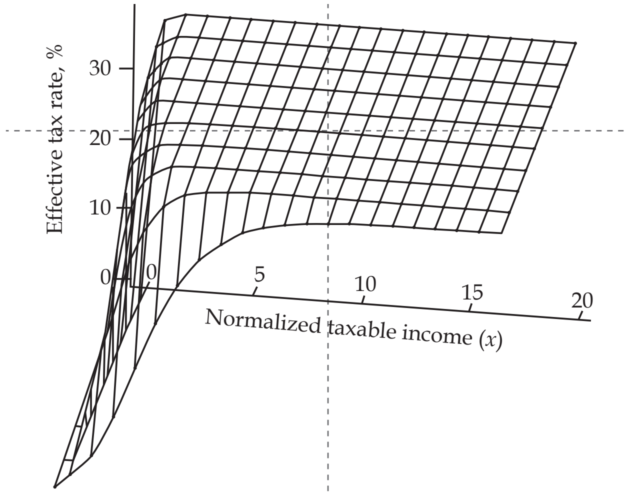

***Fig. 4.* Surface plot of the curves of function *N*(*x*) showing the effect of changes in parameters *c* and *x* while other parameters are held constant**

Source: authors' calculations

6. Conclusion

It should be noted that in our study we assumed that the distribution of taxpayers across income groups is uniform. In reality, however, there may be a lack of taxpayers in certain income ranges but this does not affect the model of a relative progression. The use of variable parameters of an antiderivative of the Gompertz function makes it a universal tool for analysis of any schedular model of progressive taxation based on a relative progression. Our model does not take into account the specific types of respondents and other parameters characteristic of any real tax system and it is not suitable for analysis of 'horizontal' equity' in income taxation.

Our study has confirmed the initial hypothesis that the application of the Gompertz function to model the key parameters of tax burden distribution in schedular progressive income taxation enables us to approximate and formalize tax burden distribution in a system reliant on a relative tax bracket-based progression.

The use of antiderivatives of the Gompertz function to describe a relative tax bracket-based progression has led us to an important conclusion concerning the application of schedules in progressive income taxation: although in schedular progressive taxation the calculations of the tax burden are simpler, such tax systems fail to provide a similar progression in the entire interval of taxpayers' income.

In practice, to overcome this drawback, we would need the data on the maximum income of taxpayers and we would also need to model progressive taxation by using coefficients which would provide the level of income above the maximum level of taxpayers' income and beyond which the speed of the tax burden's growth would start to slow down. The functional relationship between the marginal tax rate and income does not have this shortcoming but is more complicated to calculate and, therefore, its practical use would require its further adjustment to specific conditions in this or that country. Modern high-technology solutions, however, have rendered the task of tax liability calculation much less challenging and both methods of organizing progressive taxation are now practically applicable in equal measure.

The proposed model can also be used to improve the schedular income tax based on a relative tax bracket-based progression as it demonstrates the distribution of the tax burden among different income groups of taxpayers.

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