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Modeling Tax Declaration Behavior and Quality of Tax Processing: A Game Theory Approach

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ABSTRACT

Tax as one of the main levers in the micro and macro sectors of the economy it has greatly accelerated the growth of the economy, and today there are fewer countries that use it as a pillar Economic stability does not accept. On the other hand time receipt of taxes for countries is very vital and the basis of government planning for all projects and especially their budgeting. Governments are looking for ways to collect their target tax from taxpayers at the lowest possible cost. Thus, the most important step to achieve this goal is for taxpayers to declare the actual amount of tax they have paid in tax return. This paper deals with modeling the game between taxpayers and National Tax Administration. The results showed that the equilibrium declared tax of taxpayers is a function of assessed due tax, the quality of assessment groups, the number of assessments and the parameter of taxpayers' dishonesty. The taxpayers' equilibrium declared tax is increasing relative to the quality of their assessment groups and decreasing relative to other assessment groups. With increase in the likelihood of dishonesty, the declared tax of larger taxpayers will increase and the declared tax of smaller taxpayers will decrease and vice versa. Furthermore, if the quality difference of two assessment groups is only vertical, then assessed due tax and the equilibrium declared taxes will be equal. Finally, increase in the number of assessment leads to increase in the declared tax of larger taxpayers and decrease in the declared tax of smaller taxpayers and vice versa.

KEYWORDS

game theory, modeling, tax, uniform distribution, uncertainty of assessment quality

JEL C70, C63, H21, D31, D29

УДК 336.02

Моделирование поведения налогоплательщиков при декларировании доходов и качества обработки налоговых деклараций: подход теории игр

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АННОТАЦИЯ

Налогообложение как один из основных рычагов в микро- и макросекторах экономики значительно ускорило рост экономики, и сегодня стало меньше стран, которые не используют налогообложение в качестве инструмента обеспечения экономической стабильности. С другой стороны, своевременное получение налоговых доходов для стран является очень важным аспектом. Налоговые доходы являются основой для государственного планирования всех проектов и особенно их бюджетирования. Правительства ищут способы сбора налогов с налогоплательщиков при минимально возможных затратах. Наиболее важным шагом для достижения этой цели для налогоплательщиков является декларирование фактической суммы налога, которую они заплатили, в налоговой декларации. В данной статье рассматривается моделирование игры между налогоплательщиками и Национальной налоговой администрацией в процессе декларирования доходов и обработки этих деклараций. С увеличением вероятности недобросовестности задекларированный налог более крупных налогоплательщиков будет увеличиваться, а задекларированный налог более мелких налогоплательщиков будет уменьшаться, и наоборот. Кроме того, если разница в качестве двух групп оценки наблюдается только по вертикали, то начисленный налог и равновесные объявленные налоги будут равны. Результаты показали, что равновесный (равновесие по Нэшу) задекларированный налог налогоплательщиков является функцией начисленного причитающегося налога, качества оценочных групп, количества оценок и параметра недобросовестности налогоплательщиков. Равновесный объявленный налог налогоплательщиков увеличивается по отношению к качеству их оценочных групп и уменьшается по отношению к другим оценочным группам. Кроме того, увеличение количества начислений приводит к увеличению задекларированного налога более крупных налогоплательщиков и снижению задекларированного налога более мелких налогоплательщиков и наоборот.

КЛЮЧЕВЫЕ СЛОВА
теория игр, моделирование, налог, равномерное распределение, неопределенность качества оценки

1. Introduction

Today, taxes are the main source of government income and play a special role in their financing. Certainly, one of the main features of developed countries is the existence of tools and the use of efficient systems in timely tax collection and consequently reducing the cost of tax collection in the shadow of these systems. In addition, the economic and financial autonomy of the countries will be achieved through dynamic and efficient tax system; therefore, designing an optimal tax system is an important issue in economic theories' point of view. Evaluate impact and negative consequences the current corporate income tax system in the stability of the financial sector is in the focus of attention of financial economists, especially after the financial crises of recent years [1].

In recent years, due to change in the government's approach concerning the sources of revenue towards taxes, the need to identify effective barriers to the tax collection process and empowering the tax system is undeniable. One of the main barriers in achieving the goals of National Tax Administration, which is tax compliance (at the lowest cost), is the difference in the declared tax of taxpayers in the tax return and the due tax announced by the tax assessors. This underlying challenge leads to increased costs, longer collection process, and certainty of tax files. The tax system provides many incentives for people to change their taxes behavior, which means that people may decide not to declare some or all of them income and evasion of some taxes [2; 3].

Certainly, identification and resolving the deficiencies and factors that cause this difference will reduce the cost of tax collection and provide resources to the government in a timely manner and at the same time as of submitting tax return. Definitely, the best-case scenario for Tax Administration, (and of course governments) is to pay taxes at the self-assessment stage, as the higher is the voluntary tax rate in a tax system, the better will be tax indicators and the lower will be the tax gap, which leads to a move towards tax justice [4].

Kakaulina [5] and Szarowská [6] believes that taxes cause economic growth, redistribution, competitiveness of the country, performance labor market or fiscal federalism) in it will be the same time.

Also, in a study of Schiau et al. [7] it is confirmed that at proportional taxation the taxpayers are not stimulated to hide income and thus tax collection is increased [8].

Finally, it should be reminded that Tax Administration seeks to achieve a situation in which taxpayers declare and pay their actual tax amount at the stage of declaration (tax return) and in this way prevent the extension of the tax collection process (assessment, objection, reinvestigation and etc.), which takes long time and is costly for the organization and reduces the value of taxpayers' money overtime.

Governments are looking for ways to collect their target tax from taxpayers at the lowest possible cost. Thus, the most important step to achieve this goal is for taxpayers to declare the actual amount of tax they have paid in tax return. This paper deals with modeling the game between taxpayers and National Tax Administration.

This paper is organized into 5 sections. After introduction, literature review and game theory is presented respectively in the second and third parts. In the fourth part, the model used is presented in two sub-sections along with the proposed theorems, and in the fifth and final part, conclusions and recommendations are presented.

2. Literature review

The two main factors of tax evasion and tax avoidance are considered as the major challenges and obstacles in the tax collection process in most countries. Tax evasion is illegal escape from tax payment and often requires taxpayers to deliberately underrepresent their actual assets to the tax authorities in order to reduce their tax liability, usually through unreal report of their taxable assets such as income, profit, or earnings less than the actual amount [9]. On the other hand, tax avoidance is legal use of tax laws to one's own benefit to reduce tax burden. [10].

It should be noted that both tax evasion and tax avoidance can be considered as a form of tax incapacity [11]. It can also be said that tax avoidance, in its broadest sense, encompasses all arrangements for reducing, eliminating, or postponing tax debt [12].

Concerning the above definitions, one should distinguish tax evasion and tax avoidance. Tax avoidance is any legal method used by a taxpayer to minimize the amount of income tax owed or in fact to get around the law. In other words, tax avoidance is to take benefit of tax system of the country to reduce the tax owned [13]. Tax avoidance is in fact the use of gaps and weaknesses of the tax system to reduce tax without violating the laws and regulations. It should be noted that the focus of this paper is on tax avoidance. Tax planning strategies which utilize complex group structures to reduce a company's tax burden without violating tax laws may be morally reprehensible or highly questionable, as these methods are not illegal [14; 15].

According to agency theory, one of the motivations for profit management is to reduce tax liabilities and payments by minimizing the effective tax rate. Tax strategies reduce the effective tax rate either through short-term and opportunistic goals, or with the aim of reducing taxes in the long run and creating value for the company [16].

In following, some related literature will be presented. It is noteworthy that there is a bulk of studies on taxation and tax evasion, most of which dealing with the issue from the perspective of accounting (including the factors affecting the payment and etc.). Thus, hereunder, the studies focusing on taxation (whether modeling or any other type of study) and based on game theory, will be addressed.

Alm & McKee [17] examined the adaptive behavior when filing tax return for auditing purpose based on the deviation of each individual's tax report from the average of all taxpayers through laboratory tests. The results of their research showed that the tax ability can overcome the coordination of taxpayers through a slight and subtle change of the audit rule. They stated that this minor change targets audits in different ways without increasing the number of audits and assessment.

Kumacheva [18] presented a model in cooperative game between taxpayers and Tax Administration through game theory approach. In this paper, it was assumed that each taxpayer can declare his income level less than or equal to his real value. In addition, the tax and fixed fine rates and the assessor disclose the tax evasion by 100 percent. Finally, equilibrium points (Nash) for the players' behavior was obtained (by maximizing their revenue).

Abraham et al. [19] studied discriminatory tax evasion and social norms. They studied the effect of social norms and stated that theoretically and empirically it has been shown that the norm of tax compliance has a negative and strong (significant) effect on its amount and tax evasion is independent of it.

Cerqueti & Copier [20] studied the relation between corruption and tax evasion in the environmental policymaking. To this end, they designed a game of incomplete information, in which the government could have a strategy of its choice in two ways. The results indicated that in a highly motivated country, the motivational channel is more effective than obedience.

Sokolovskyi [21] presented a theoretical model for tax evasion game through analyzing the interaction of factors and optimizing tax burden (the problem of optimizing the real tax burden). The results show that in the presented curve, the dependency of the actual and declared tax burden is not in one point (like Laffer curve); rather, there are three relative maximum points.

Kiral & Mavruk [22] studied the paid tax of big companies through game theory approach. They considered an application of mixed strategy for unlimited iterative games, in which the company pays its tax in four payouts. One of their most important results was that the solutions sets are linear zero-sum game and variable trapezoidal sum, and to prevent tax evasion, the number of inspectors and audits should be increased.

Gubar et al. [23] examined network games and structures on corruption, income inequality, and tax control. They sought to present a model in which taxpayers (in rich and poor groups, who all pay tax) decide whether to pay tax or not concerning their personal income and preferences, as well as the audit and tax control data. Their results showed that taxpayers' initial and final preferences depend on important parameters such as tax rates, fines, audit information and costs.

Chica et al. [24] presented an evolutionary game to understand the dynamics of consumption tax fraud. They claimed that the tax paid by each player depends on the amount of tax paid (more or less) and the likelihood of subjective inspection by tax officials. Finally, they showed that increase in the likelihood of subjective auditing is more efficient for low-volume trades than for high-volume trades. Moreover, the results of their studies indicated that social rewards for those who cooperate in tax payment and alternative penalties for those who evade taxes could be effective policies, although its success depends on the distribution of audit probability for different types of transactions (small or large).

The investigations show that a lot of research has dealt with taxation (and to a very small extent, game theory); however, as it turns out, there is a limited number of studies focusing on game theory and presenting a model. On the other hand, no model has been so far presented with consideration of the utility functions used in this research. Furthermore, the existing studies have not addressed the important issues of equilibrium declared tax, optimal number of assessments, collection function of the Tax Administration and the quality of the assessing groups, that are indeed the most important issues in the game between the Tax Administration and taxpayers. Therefore, the innovation of this study is consideration of these important cases and presenting several theorems by obtaining Nash equilibrium points.

3. Game theory

Game theory is the study of methods in which the mutual selection of economic players based on their preferences, produces some results that may not be intended by any of them. Game theory utilizes mathematical models to analyze the methods of cooperation or competition of rational and intelligent beings and tries to model the mathematical behavior governing a strategic situation (conflict of interest) [25]. This situation arises when a person's success depends on the strategies that others choose. The ultimate goal of this knowledge is to find the optimal strategy for players [26].

Some researchers compare the importance of game theory design to the discovery of the double DNA spirals and often refer to it as "a theory that can explain everything" [27].

If the number of players (agents) in opposition is limited, game theory can be very useful because in this case the behavior of each player has a significant effect on the income of other players [28]. It should also be noted that game theory allows model makers to think the same as economists when price theory cannot be accountable [29]. Game theory has evolved due to the continuous efforts of many social scientists, especially economics and pure sciences (mathematics and statistics) and today, as one of the most important achievements of human knowledge, serves various sciences including humanities, natural, technical and pure sciences [30]. Game theory is now very widely used throughout the profession and has become a major tool for the construction of new economic models [31].

The main principle of game theory is that all players in a common game have common knowledge. In other words, all players in a game know the structure of the game, as well they know that their competitors also know it, and at the same time they know that other competitors know that they know this, and so on.

One of the most common types of games is static games of complete information, in which players choose their strategy simultaneously, and every player is fully aware of what other players achieve in the game. In static games with complete information, each player chooses his strategy with full awareness of the interests, but not the choices, of the rival player; in other words, players choose their strategy at the same time [32].

The basic assumption of these games is that each side of the game does not know the choice of the other side (opponent) and, in fact, it seems as if each makes their choice at the same time. Another basic assumption in these games is that all consequences of the game are known to all players, i.e. each player knows what he will gain in return for his own choice and his competitor's. Most games in the real world are static games. The equilibrium resulting from this type of games is called Nash equilibrium, which is defined as follow:

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i}).$$

That is, the player's strategy is the best reaction to the selected act of other competitors [33].

4. Modeling and simulation results

Suppose taxpayers are evenly distributed in the range [0, 1]. After paying his due book tax, a taxpayer located at point on the said interval achieves a surplus of Christou & Vettas [34]:

$$u(w, i) = R - \frac{1}{1 + r\theta}(w - T_i)^2 + q_i - E_i. \quad (1)$$

Where R is the reservation value of sale (products or services and etc.), which is assumed to be high enough so that all taxpayers pay taxes; in other words, the market is fully covered.

$$\frac{1}{1 + r\theta}$$

is called probability of non-disclosure (violation) and it means that the taxpayer may not disclose all the facts related to his payment. Here, by r, it is meant an assessment performed by the tax assessment groups, such that the higher is r, the less likely will be non-disclosure of violation ($r \geq 0$).

Furthermore, by θ, it is meant dishonesty of the taxpayers such that the higher is θ, the less likely will be non-disclosure of violation ($\theta \geq 0$). w is the taxpayer situation and T_i is the due tax for taxpayer i. q_i is the quality of assessment groups i (it is assumed that the difference in the quality of assessors is unknown to the taxpayers) and E_i is the declared amount by taxpayer i.

This equation (1) shows that the assessment groups are different both horizontally and vertically. In order to obtain the effect of the uncertainty of the quality of the assessment groups on the taxpayers' declared tax, it is assumed that q_i is a random value, which is unknown to the taxpayer at the time of declaration. In this case, the game will be as follows:

1. The difference in the quality of the assessment groups ($q_i - q_j$) is obvious and as common knowledge.

2. The taxpayers simultaneously select their declared tax.

The Tax Administration seeks to maximize its expected revenue, and on the other hand, the taxpayer seeks to maximize its net surplus after tax payment. This idea shows that when the quality of the assessment groups is known to taxpayers, the change in the procedure of the Tax Administration is very costly [35]. It is assumed that the difference in the quality of the assessment groups ($q_i - q_j$), which is random, is uniformly distributed over the interval

$$\left[-\frac{1}{2}, \frac{1}{2}\right].$$

It should be noted that since the quality difference is a random value, the utility assigned to each taxpayer will be random in respect to each declared tax.

Assume that the difference in the quality of two assessment groups ($q_i - q_j$) which is random is in three H_2, S, H_1 states. H_1 means that the difference in quality of two assessment groups is very high and the quality of assessment group 1 is much better than that of assessment group 2. S shows the state in which the difference in quality of two assessment groups is minor and the taxpayers do not clearly prefer the quality of neither assessment groups over the other.

Furthermore, H_2 means that the difference in quality of two assessment groups is very high and the quality of assessment group 2 is much better than that of assessment group 1. In addition, suppose that the difference in quality of two assessment groups is in interval

$$\left[-\frac{1}{2}, -\frac{1}{4}\right],$$

equal to H_1, if the difference in quality of two assessment groups is in interval

$$\left[-\frac{1}{4}, \frac{1}{4}\right],$$

equal to S, and finally if the difference in quality of two assessment groups is in interval

$$\left[\frac{1}{4}, \frac{1}{2}\right],$$

equal to H_2, now, we obtain the equilibrium declared tax through inverse inference.

4.1. Equilibrium declared tax

As far as assessment groups 1 and 2 can have a situation on the line, for simplicity purpose, we assume that the assessment group 1 is in the left side of assessment group 2 ($T_1 \leq T_2$). That means that the due tax of assessment group 1 is less than that of assessment group 2 (assessment group 2 deals with larger taxpayers). Now, knowing the due tax of taxpayers, the location of the indifferent taxpayer should be found out.

The indifferent taxpayer is one who does not care which assessment group sets tax for him, as he has accurately declared its due tax and if it is assessed by either assessment group, the amount of due tax will be the same as his declared tax.

Suppose z is the number of taxpayers assessed by Group 1; therefore, z-1 will be the number of taxpayers assessed by Group 2, ($z \in (0, 1)$). The number of taxpayers assessed by two assessment groups will be obtained by obtaining the position of the indifferent taxpayer in respect to assessment groups 1 and 2. Therefore, concerning equation 1 and as the indifferent taxpayer is located at point z, we have:

$$E_1 + \frac{1}{1+r\theta}(z-T_1)^2 = \\ = -q + E_2 + \frac{1}{1+r\theta}(z-T_2)^2. \quad (2)$$

Where $q = q_2 - q_1$. It is clear that q can be positive, negative or zero depending on the quality of the assessment groups.

The position of the indifferent taxpayer depends on whether it is declared or due tax and on the quality of the assessment groups. Therefore $z = z(E_1, E_2, T_1, T_2, q)$. By solving equation 2 and as per $z \in (0, 1)$, we have:

$$z^* = \frac{T_1 + T_2}{2} + \frac{(r\theta + 1)(E_2 - q - E_1)}{2(T_2 - T_1)}.$$

Thus, the income function of assessment groups will be as:

$$I_1 = E_1 z, \quad I_2 = E_2(1 - z). \qquad (3)$$

While:

$$z = \begin{cases} 0 \quad if \quad \dfrac{T_1 + T_2}{2} + \dfrac{E_2 - q - E_1}{2(T_2 - T_1)} \le 0 \\ \dfrac{T_1 + T_2}{2} + \dfrac{(r\theta + 1)(E_2 - q - E_1)}{2(T_2 - T_1)} \; if \\ if \; \dfrac{T_1 + T_2}{2} + \dfrac{E_2 - q - E_1}{2(T_2 - T_1)} \in (0,1) \\ 1 \quad if \quad \dfrac{T_1 + T_2}{2} + \dfrac{E_2 - q - E_1}{2(T_2 - T_1)} \ge 1 \end{cases} \qquad (4)$$

These equations are obtained concerning uniform distribution of consumers. Now, the equilibrium declared tax will be extracted:

Theorem 1. *The equilibrium declared tax of taxpayers 1 and 2 in respect to $0 \le T_1 \le T_2 \le 1$ will be*:

$$E_1^* = \begin{cases} \dfrac{-q - T_1^2 + 2T_1 + T_2^2 - 2T_2}{(r\theta + 1)} \; if \; q < -\dfrac{1}{4} \\ \dfrac{-q(r\theta + 1) - T_1^2 - 2T_1 + T_2^2 + 2T_2}{3(r\theta + 1)} \; if \\ if \;\; q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], \; 0 < z < 1 \\ 0 \qquad if \qquad q > \dfrac{1}{4} \end{cases} \qquad (5)$$

$$E_2^* = \begin{cases} 0 \qquad if \qquad q < -\dfrac{1}{4} \\ \dfrac{q(r\theta + 1) + T_1^2 - 4T_1 - T_2^2 + 4T_2}{3(r\theta + 1)} \; if \\ if \; q \in [-\dfrac{1}{4}, \dfrac{1}{4}] \;\; , \;\; 0 < z < 1 \\ q - \dfrac{T_2^2 - T_1^2}{r\theta + 1} \qquad if \qquad q > \dfrac{1}{4} \end{cases} \qquad (6)$$

Proof. To prove this theorem, just equation 4 should be inserted in equation 3.

$$q < -\frac{1}{4}$$

shows that the quality of assessment group 1 is high enough; therefore, assessment group 1 will be the only assessment group.

$$q > \frac{1}{4}$$

shows that the quality of assessment group 2 is high enough; therefore, assessment group 2 will be the only assessment group. Other states are for cases where the difference in the assessment quality of two groups is minor and depends on their position.

In addition, it is possible to refer to other important results from this theorem. If the quality difference between two assessment groups is only horizontal, then $q = 0$ and it is possible to simply obtain equilibrium declared taxes, which are:

$$E_1^* = \frac{-T_1^2 - 2T_1 + T_2^2 + 2T_2}{3(r\theta + 1)},$$

$$E_2^* = \frac{T_1^2 - 4T_1 - T_2^2 + 4T_2}{3(r\theta + 1)}.$$

Furthermore, if the quality difference of two assessment groups is only vertical, then $T_1 = T_2$ and the equilibrium declared taxes will be equal to $(q / 3)$. Moreover, concerning the results, the equilibrium declared tax of taxpayers 1 and 2 based on the quality difference of the assessment groups will be equal $(E_1^* = E_2^*)$ where:

$$\begin{cases} q = -T_1^2 + 2T_1 + T_2^2 - 2T_2 \; if \; q < -\dfrac{1}{4} \\ q = -\dfrac{T_1^2 - T_1 - T_2^2 + T_2}{r\theta + 1} \; if \\ if \;\; q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], \; 0 < z < 1 \\ q = \dfrac{T_2^2 - T_1^2}{r\theta + 1} \qquad if \qquad q > \dfrac{1}{4} \end{cases}$$

Theorem 2. *The taxpayers' equilibrium declared tax is increasing relative to the quality of their assessment groups and decreasing relative to the other assessment groups.*

Proof. Before proving this theorem, it should be noted that as mentioned earlier, taxpayers before the indifferent taxpayer are assumed to be less inclined to pay, and taxpayers after the indifferent taxpayer tend to pay higher (larger taxpayers). As previously mentioned, assessment group 1 is the one with lower due tax (as they deal with smaller taxpayers or those with tendency to pay less) and assessment group 2 is the one with higher due tax (as they deal with larger taxpayers or those with tendency to pay more). In this case, concerning equations 5 and 6, we have:

$$\begin{cases} \dfrac{\partial E_1^*}{\partial q_1} = \dfrac{1}{r\theta+1} & q < -\dfrac{1}{4}, \\ \dfrac{\partial E_1^*}{\partial q_1} = \dfrac{1}{3} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right] \\ \dfrac{\partial E_1^*}{\partial q_2} = -\dfrac{1}{r\theta+1} & q < -\dfrac{1}{4}, \\ \dfrac{\partial E_1^*}{\partial q_2} = -\dfrac{1}{3} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right] \end{cases}$$

$$\begin{cases} \dfrac{\partial E_2^*}{\partial q_1} = -\dfrac{1}{3} \; q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], & \dfrac{\partial E_2^*}{\partial q_1} = -1 \; q > \dfrac{1}{4} \\ \dfrac{\partial E_2^*}{\partial q_2} = \dfrac{1}{3} \; q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], & \dfrac{\partial E_2^*}{\partial q_1} = 1 \; q > \dfrac{1}{4} \end{cases}$$

The results clearly show how each group of the taxpayers behave with the assessment groups of their own or others. In other words, the declared tax of taxpayers directly depends on the quality of their assessment groups and has inverse relation with the quality of the other assessment groups.

Theorem 3. *With increase in the number of assessments (r), the declared tax of larger taxpayers will increase and the declared tax of smaller taxpayers will decrease and vice versa.*

Proof. To prove this theorem, it is sufficient to take derivative of equations 5 and 6 with respect to the number of assessments and simplify the results. Therefore, we have:

$$\begin{cases} \dfrac{\partial E_1^*}{\partial r} = \dfrac{\theta(T_1^2 - 2T_1 - T_2^2 + 2T_2 + q)}{(r\theta+1)^2} & q < -\dfrac{1}{4}, \\ \dfrac{\partial E_1^*}{\partial r} = \dfrac{\theta(T_1^2 + 2T_1 - T_2^2 - 2T_2)}{3(r\theta+1)^2} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right] \quad (a) \\ \dfrac{\partial E_2^*}{\partial r} = -\dfrac{\theta(T_1^2 - 4T_1 - T_2^2 + 4T_2)}{3(r\theta+1)^2} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], \\ \dfrac{\partial E_2^*}{\partial r} = \dfrac{\theta(T_2^2 - T_1^2)}{(r\theta+1)^2} & q > \dfrac{1}{4} \quad (b) \end{cases}$$

As it is clear, concerning Equation 7 and as all values are positive, the whole fraction (given that $T_1 < T_2$) becomes negative. Similarly, in equation B, it is clear that the whole fraction is positive. According to the results of this theorem, the Tax Administration can apply the necessary policies to conduct more assessment for larger taxpayers to reduce the tax costs (through reducing assessment and as a result reducing costs in board assessment stages and etc.) through increasing taxpayers' declared tax (and of course, reducing the difference between declared and due book taxes) and, in the long run, acceptance of the declared tax of taxpayers. It should be noted that these results are consistent with the results of Kiral & Mavruk [22].

Here, it should be pointed out that the results of this theorem are true for the likelihood of taxpayers' dishonesty (θ) in the same way and the result is as follows:

$$\begin{cases} \dfrac{\partial E_1^*}{\partial r} = \dfrac{r(T_1^2 - 2T_1 - T_2^2 + 2T_2 + q)}{(r\theta+1)^2} & q < -\dfrac{1}{4}, \\ \dfrac{\partial E_1^*}{\partial r} = \dfrac{r(T_1^2 + 2T_1 - T_2^2 - 2T_2)}{3(r\theta+1)^2} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right] \\ \dfrac{\partial E_2^*}{\partial r} = -\dfrac{r(T_1^2 - 4T_1 - T_2^2 + 4T_2)}{3(r\theta+1)^2} & q \in \left[-\dfrac{1}{4}, \dfrac{1}{4}\right], \\ \dfrac{\partial E_2^*}{\partial r} = \dfrac{r(T_2^2 - T_1^2)}{(r\theta+1)^2} & q > \dfrac{1}{4} \end{cases}$$

Here, with increase in the likelihood of dishonesty (θ), the declared tax of larger taxpayers will increase and the declared tax of smaller taxpayers will decrease and vice versa. This result can be interpreted as, the higher is the likelihood of the larger taxpayer's dishonesty, (because the larger taxpayer is better known), the more will be the reasons for assessment and the higher will be the likelihood of detecting corruption.

In follow, the income function (earning) of the Tax Administration will be extracted and the results interpreted.

4.2. Income function (earning) of tax administration

In this part, knowing the equilibrium declared taxes, we seek to find maximum income/revenue of Tax Administration. On the other hand, taxpayers seek to maximize their expected profit. The expected income (earning) of Tax Administration is shown by which is random based on the difference in quality of two assessment groups. In this condition, the expected income function of assessment groups 1 and 2 will be as:

$$EI_1(T_1,T_2) = = \int_{-\frac{1}{2}}^{-\frac{1}{4}} I_1^m(T_1,T_2)dF + \int_{-\frac{1}{4}}^{\frac{1}{4}} I_1^c(T_1,T_2)dF, \quad (7)$$

$$EI_2(T_1,T_2) = = \int_{-\frac{1}{4}}^{\frac{1}{4}} I_2^c(T_1,T_2)dF + \int_{\frac{1}{4}}^{\frac{1}{2}} I_2^m(T_1,T_2)dF. \quad (8)$$

Where

$$F(T) = \frac{2T+1}{2}$$

is cumulative distribution function of parameter q which is evenly distributed in

$$\left[-\frac{1}{2}, \frac{1}{2}\right].$$

It should be noted that depending on the difference in the quality of assessment, both angular and internal solutions may occur. $I_1^m(T_1,T_2)$ show the state, which is only assessed by assessment group 1 and $I_1^c(T_1,T_2)$ shows the expected income of the assessment group 1; while both assessment groups 1 and 2 do assessment. Now, the expected income of two assessment groups can be obtained.

Theorem 4. *The expected income of two assessment groups will be as:*

$$I_1 = -\frac{r\theta+1}{1728(T_1-T_2)} - \frac{\begin{pmatrix} 8T_1^3 + 8T_1^2(T_2+13) - 8T_1(T_2^2+14) - \\ -8T_2^3 - 104T_2^2 + 112T_2 - 27 \end{pmatrix}}{288(r\theta+1)}, \quad (9)$$

$$I_2 = -\frac{r\theta+1}{1728(T_1-T_2)} - \frac{\begin{pmatrix} 8T_1^3 + 8T_1^2(T_2-17) + 8T_1(16-T_2^2) - \\ -8T_2^3 + 136T_2^2 - 128T_2 - 27(r\theta+1) \end{pmatrix}}{288(r\theta+1)}. \quad (10)$$

Proof. By inserting the equilibrium declared taxes from equations 5 and 6 in profit function (equation 3) we have:

$$I_1^m(T_1,T_2) = \frac{-q - T_1^2 + 2T_1 + T_2^2 - 2T_2}{(r\theta+1)},$$

$$I_1^c(T_1,T_2) = = \frac{(-q(r\theta+1) - T_1^2 - 2T_1 + T_2(T_2+2))^2}{18(T_2-T_1)(r\theta+1)}.$$

In the same way, the expected income function of assessment group 2 will be as:

$$EI_2(T_1,T_2) = = \int_{-\frac{1}{4}}^{\frac{1}{4}} I_2^c(T_1,T_2)dF + \int_{\frac{1}{4}}^{\frac{1}{2}} I_2^m(T_1,T_2)dF.$$

Where

$$I_2^m(T_1,T_2) = q - \frac{T_2^2 - T_1^2}{r\theta+1},$$

$$I_2^c(T_1,T_2) = = \frac{(q(r\theta+1) + T_1^2 - 4T_1 - T_2^2 + 4T_2)^2}{18(T_2-T_1)(r\theta+1)}.$$

Therefore, knowing expected income functions of assessment groups 1 and 2 in different intervals and then applying them in equations 7 and 8, the results will be as follow:

$$I_1 = -\frac{r\theta+1}{1728(T_1-T_2)} - \frac{\begin{pmatrix} 8T_1^3 + 8T_1^2(T_2+13) - 8T_1(T_2^2+14) - \\ -8T_2^3 - 104T_2^2 + 112T_2 - 27 \end{pmatrix}}{288(r\theta+1)},$$

$$I_2 = -\frac{r\theta+1}{1728(T_1-T_2)} - \frac{\begin{pmatrix} 8T_1^3 + 8T_1^2(T_2-17) + 8T_1(16-T_2^2) - \\ -8T_2^3 + 136T_2^2 - 128T_2 - 27(r\theta+1) \end{pmatrix}}{288(r\theta+1)}.$$

Theorem 5. *The expected income of two investment groups is equal when:*

$$r = -\frac{80(T_1^2 - T_1 - T_2^2 + T_2)}{9\theta}$$

or

$$\theta = -\frac{80(T_1^2 - T_1 - T_2^2 + T_2)}{9r}.$$

Proof. To prove this theorem, just equations 9 and 10 should be put equal and the equation should be obtained in respect to r and θ. In addition, if assuming that $T_2 = \alpha T_1$, where $\alpha > 1$ (since as previously mentioned, the due tax of group 2 is higher than group 1), then, concerning the results, it will be clear that the number of assessments is decreasing in respect to due tax of group 1 and increasing in respect to due tax of group 2.

5. Conclusion

The difference in the declared tax of taxpayers in tax return and the due task determined by tax assessors, is one of the main obstacles in achieving the goals of National Tax Administration, which is tax compliance (at the lowest cost). This underlying challenge leads to increased costs, longer collection process, and certainty of tax files.

Therefore, Tax Administration seeks to achieve a situation in which taxpayers declare and pay their actual tax amount at the stage of declaration (tax return) and in this way prevent the extension of the tax collection process (assessment, objection, reinvestigation and etc.), which is costly for organization in terms of time and money and reduces the value of taxpayers' assets overtime.

This paper has dealt with modeling the game between taxpayers and National Tax Administration. The results showed that the equilibrium declared tax of taxpayers is a function of due tax, the quality of assessment groups, the number of assessments and the parameter of taxpayers' lack of honesty.

The taxpayers' equilibrium declared tax is increasing relative to the quality of their assessment groups and decreasing relative to the other assessment groups. Furthermore, increase in the number of assessments leads to increase in the declared tax of larger taxpayers and decrease in the declared tax of smaller taxpayers and vice versa. On the other hand, the number of assessments is increasing in respect to due tax of group 1 and increasing in respect to group 2.

References

1. Toshmatov S.A., Abdullaev Z.A., Ahrorov Z.O. Influence of Corporate Income Tax to Loan Loss Provision: Evidence from Uzbekistan. *Journal of Tax Reform*. 2022;8(3):236–250. https://doi.org/10.15826/jtr.2022.8.3.119

2. Wang K.Y., Zhang L. Underground economy and GDP growth: Evidence from China's tax reforms. *Journal of Tax Reform*. 2021;7(1):87–107. https://doi.org/10.15826/jtr.2021.7.1.092

3. Gorecki A.M., Letki N. (2021). Social Norms Moderate the Effect of Tax System on Tax Evasion: Evidence from a Large-Scale Survey Experiment. *Journal of Business Ethics*. 2021;172:727–746. https://doi.org/10.1007/s10551-020-04502-8

4. Yeganeh H.Y., Sarveolia, M.H.E., Alasvand F., Delvar A. Importance of Factors Influencing Intention-to-pay Tax from the Perspective of Tax Payers: (Case Study: Taxpayers of Iranian National Tax Admission Organization-Tehran). *Journal of Management Accounting and Auditing Knowledge*. 2019;8(29):203–214. Available at: https://jmaak.srbiau.ac.ir/article_13845.html?lang=en

5. Kakaulina O.M. Projected shortfall in personal income tax revenues of regional governments in Russia due to the COVID-19 pandemic. *Journal of Tax Reform*. 2021;7(1):39–54. https://doi.org/10.15826/jtr.2021.7.1.089

6. Szarowská I. Personal income taxation in a context of a tax structure. *Procedia Economics and Finance*. 2014;12:662–669. https://doi.org/10.1016/S2212-5671(14)00391-8

7. Laura-Liana S., Carmen M.A. The flat Tax Effects – Theoretical and Empirical Evidence in Western and Eastern European countries. *Annals of Faculty of Economics*. 2009;3(1):343–348. Available at: https://ideas.repec.org/a/ora/journl/v3y2009i1p343-348.html

8. Tanchev S. Long-run equilibrium between personal income tax and economic growth in Bulgaria. *Journal of Tax Reform*. 2021;7(1):55–67. https://doi.org/10.15826/jtr.2021.7.1.090

9. Becker G.S. Crime and Punishment: an Economic Approach. In: Fielding N.G., Clarke A., Witt R. (eds). *The Economic Dimensions of Crime*. London: Palgrave Macmillan; 1968. https://doi.org/10.1007/978-1-349-62853-7_2

10. Dyreng S.D., Hanlon M., Maydew E.L. The Effects of executives on corporate tax avoidance. *The Accounting Review*. 2010;85(4):1163–1189. https://doi.org/10.2308/accr.2010.85.4.1163

11. Wenzel M. The Impact of Outcome Orientation and Justice Concerns on Tax Compliance. *Journal of Applied Psychology*. 2002;87(4):629–645. https://doi.org/10.1037/0021-9010.87.4.629

12. Freedman J. Defining Taxpayer Responsibility: In Support of a General Anti-Avoidance Principle. *British Tax Review*. 2004:4:332–357. Available at: https://ssrn.com/abstract=900043

13. Dyreng S.D., Hanlon M., Maydew E.L. Long-Run Corporate Tax Avoidance. *The Accounting Review*. 2008;83(1):61–82. https://doi.org/10.2308/accr.2008.83.1.61

14. Wilson R.J. An Examination of Corporate Tax Shelter Participants. *The Accounting Review*. 2009;84(3):969–999. https://doi.org/10.2308/accr.2009.84.3.969

15. Lisowsky P. Seeking Shelter: Empirically Modeling Tax Shelters Using Financial Statement Information. *The Accounting Review*. 2010;85(5):1693–1720. https://doi.org/10.2308/accr.2010.85.5.1693

16. Vaez S.A., Daresh F., Basirat M., Kaeb Amir A. Evaluating Tax Persistence and Future Tax Risk of Firms by Emphasizing on Ownership Type. *Journal of Tax Research*. 2019;26(40):187–206. Available at: http://taxjournal.ir/article-1-1596-en.html

17. Alm J., McKee M. Tax compliance as a coordination game. *Journal of Economic Behavior & Organization*. 2004;54(3):297–312. https://doi.org/10.1016/j.jebo.2003.02.003

18. Kumacheva S.S. A Game Theoretical Model of Interaction Between Taxpayers and the Tax Authority. *Contributions to Game Theory and Management*. 2010;3:257–266. Available at: https://www.mathnet.ru/links/ffaa0acdc6d8880e0a60e118b0c20d5e/cgtm89.pdf

19. Wrede M., Abraham M., Lorek K., Richter F. *Collusive Tax Evasion and Social Norms*. CESifo Working Paper, No. 5167, 2015. Available at: http://hdl.handle.net/10419/112859

20. Cerqueti R., Copier R. Corruption, Evasion and Environmental Policy: A Game Theory Approach. *IMA Journal of Management Mathematics*. 2016;27(2):235–253. https://doi.org/10.1093/imaman/dpu019

21. Sokolovskyi D. *Game-theoretic model of tax evasion: analysis of agents' interaction and optimization of tax burden*. MPRA Paper, 2018. Available at: https://mpra.ub.uni-muenchen.de/id/eprint/86415

22. Kiral E., Mavruk C. Corporate Tax Payoff in A Game Theory Approach. *Ege Academic Review*. 2018;18(2):195–215. https://doi.org/10.21121/eab.2018237350

23. Gubar E., Carrera E.J., Kumacheva S., Zhitkova E., Tomilina G. Games and network structures on corruption, income inequality, and tax control. *Contributions to Game Theory and Management*. 2020;13:152–172. https://doi.org/10.21638/11701/spbu31.2020.09

24. Chica M., Hernandez J., Manrique-de-Lara-Penate C., Chiong R. An Evolutionary Game Model for Understanding Fraud in Consumption Taxes. *IEEE Computational Intelligence Magazine*. 2021;16(2):62–76. https://doi.org/10.1109/MCI.2021.3061878

25. Owen G. Game Theory. In: Chadwick R. (ed.) *Encyclopedia of Applied Ethics (Second Edition)*. Elsevier; 2012, pp. 391–398. https://doi.org/10.1016/B978-0-12-373932-2.00178-2

26. Shahbazi K., Salimian S. The Expansion of Location Theories of Firms and Products' Consistency Using Triangular Distribution Approach. *Iranian Economic Review*. 2017;21(3):497–518. https://doi.org/10.22059/ier.2017.62937

27. Varoufakis Y. Can it Unify the Social Sciences? *Organization Studies*. 2008;29(8-9):1255–1277. https://doi.org/10.1177/0170840608094779

28. Shy O. *Industrial organization: theory and applications*. Cambridge: MIT Press; 1995. Available at: https://mitpress.mit.edu/9780262691796/industrial-organization/

29. Gibbons R. An Introduction to Applicable Game Theory. *Journal of Economic Perspective*. 1997;11(1):127-149. https://doi.org/10.1257/jep.11.1.127

30. Maskin E. *Recent Developments in Game Theory*. Edward Elgar Pub.; 1999. 490 p.

31. Khalilzade S.Z., Salimian S., Shahriyari I., Pishani Y.H., Salimian S. Economic analysis of tax installment forgiveness using game theory approach: The case of Iran. *Journal of Sustainable Business, Economics and Finance*. 2022;1(3):1–15. https://doi.org/10.31039/josbef.2022.1.3.24

32. Osborne M.J., Rubinstein A. *A course in game theory*. MIT press; 1994. Available at: https://arielrubinstein.tau.ac.il/books/GT.pdf

33. Mas-Colell A., Whinston M.D., Green J.R. *Microeconomic Theory*. London: New York: Oxford University Press; 1995. 977 p. Available at: http://englishonlineclub.com/pdf/Microeconomic%20Theory%20[EnglishOnlineClub.com].pdf

34. Christou Ch., Vettas N. Location choices under quality uncertainty. *Mathematical Social Sciences*. 2005;50(3):268–278. https://doi.org/10.1016/j.mathsocsci.2005.05.002

35. Salimian S., Shahbazi K., Khodadad Kashi F. (2017). Localization of firms in conditions of triangular distribution of consumers and uncertain quality of firms. *Iranian Industrial Economic Studies*. 2017;1(1):13–20. https://doi.org/10.30466/iies.2018.20529

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